Monte Carlo simulations of light transport

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http://omlc.org
Oregon Medical Laser Center
Find value of $\pi$

787 points within circle
213 points outside circle
1000 points total

x = rand(1000,1);
y = rand(1000,1);
\[
\frac{787}{1000} = \frac{\text{area of circle}}{\text{area of box}} = \frac{\pi \left(\frac{w}{2}\right)^2}{w^2} = \pi / 4 \quad \rightarrow \quad \pi = 4 \frac{787}{1000} = 3.148
\]

\[N = 1000; \quad \% \text{ total number of points}\]
\[w = 1; \quad \% \text{ arbitrary diameter of circle}\]
\[x = w \ast \text{rand}(N,1); \quad \% \text{ random number generator}\]
\[y = w \ast \text{rand}(N,1);\]
\[r = \sqrt{(x-w/2)^2 + (y-w/2)^2};\]
\[\text{ncirc} = \text{sum}(r \leq w/2); \quad \% \# \text{ points within circle}\]
\[\text{ratio} = \text{ncirc} / N \quad \% \# \text{ within circle} / \text{ total}\]
\[\text{calc}_\pi = 4 \ast \text{ratio}\]

\[\pi = (4)(787/1000) = 3.148\]

True value = 3.1415
Find value of e

640 points below p(x) line
360 points above p(x) line
1000 points total

x = rand(1000,1);
y = rand(1000,1);
\[
\frac{640}{1000} = \frac{\text{# below } p(x)}{\text{total #}} = \frac{1000 \int_0^1 e^{-x} \, dx}{1000} = 1 - \frac{1}{e} \quad \rightarrow \quad e = \frac{1}{1 - 0.64} = 2.67
\]

N = 1e3; % total # of points

\[
x = \text{rand}(N,1);
y = \text{rand}(N,1);
px = \exp(-x);
npx = \text{sum}(y<=px); \quad \% \# \text{ points under } p(x)
\]

\[
\text{ratio} = npx/N; \quad \% \text{ ratio of areas}
\]

\[
\% \text{ ratio} = 1 - 1/e
\]

\[
\text{answer} = 1/(1-\text{ratio})
\]

\[
\text{ratio} = 640/1000 = 0.64
\]

\[
e = 1/(1-0.64) = 2.67
\]

True value = 2.718
Probability density function \( p(x) \)

Bucket method

bucket of marbles

marbles mimicking \( p(x) \)
Analytic method

\[ RND = F(x_1) \]

\[ x_1 = \frac{-\ln(RND)}{\mu} \]
Launch photons uniformly over a circular area

$$RND = \frac{r_1^2}{a^2}$$

$$r_1 = a \sqrt{RND}$$

$$x = r \cos(\phi)$$
$$y = r \sin(\phi)$$

where $$\phi = RND2\pi$$

$$F(r_1) = \int_0^{r_1} \frac{2r}{a^2} dr = \frac{r^2}{a^2} |_{r_1} = \frac{r_1^2}{a^2}$$

$$p(r) = \frac{1}{\pi a^2} 2\pi r = \frac{2r}{a^2}$$

$$\frac{1}{\pi a^2} \int_0^a 2\pi r dr = 1$$
\[ N = 1000; \]
\[ a = 1; \]
\[ rnd = \text{rand}(N,1); \]
\[ r1 = a*\sqrt{\text{rnd}}; \]
\[ \phi = \text{rand}(N,1)*2*\pi; \]
\[ x = r1.*\cos(\phi); \]
\[ y = r1.*\sin(\phi); \]
j = 4;
tissue(j).name = 'reticular.dermis';
B = 0.001;  % blood content (whole blood = 150gHGb/liter)
S = 0.075;  % oxygen saturation of hemoglobin
W = 0.55;   % water content
M = 0;      % melanosome volume fraction in epidermis
musp500 = 30;  % reduced scattering at 500 nm, mus(1-g)
fray = 0.62; % fraction Rayleigh scattering at 500 nm
bmie = 0.91; % = 1-fray, Mie scattering at 500 nm
g = 0.90;   % anisotropy (g_1 = forward vs backward balance)
musp = musp500*(fray*(nm/500).^-4 + (1-fray)*(nm/500).^-bmie);
X = [B*S B*(1-S) W M]';
tissue(j).mua = MU*X;  % matrix multiplication
tissue(j).mus = musp/(1-g);
tissue(j).g = g;
Optics of Sea Coral

Daniel Wangspraseurt
Phototherapy of the knee to reduce inflammation

mcxyz.c
Monte Carlo code
https://omlc.org/software/mc/

mcml.c

“Monte Carlo Multi-Layered”
Planar layers of tissue, each with distinct optical properties.

mcxyz.c

https://omlc.org/software/mc/mcxyz/
Multi voxels, each voxel with distinct optical properties.